



# Wave Propagation Properties in $\text{LiNbO}_3$ Using the Finite Element Method

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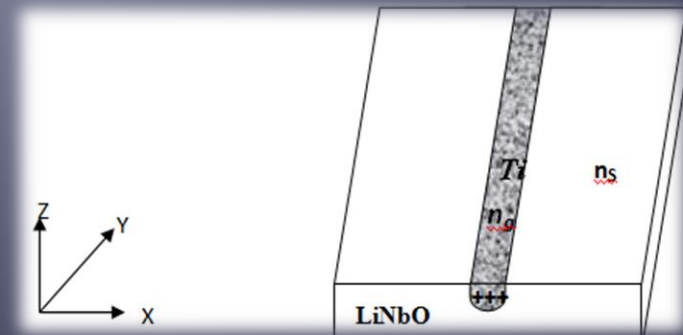
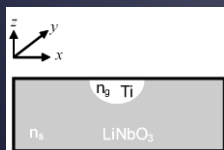
## Outline:

- ❑ The objective of this work is to study light propagation in optical waveguides of the type Ti : LiNbO<sub>3</sub> using the finite element method.
- ❑ The variational principle use in the determination of the orthogonal mode sizes for monomode titanium-in-diffused LiNbO<sub>3</sub> strip waveguides.
- ❑ A comparison between the results obtained with published data.
- ❑ Finite element method results in 2D and 3D.
- ❑ The results obtained here are compared to those recently published

## Introduction

- ✓ Nowadays, an increasing attention is being given to strip waveguides formed by the diffusion of titanium (Ti) in lithium niobate ( $\text{LiNbO}_3$ ). Such waveguides provide the basis for many promising devices for both optical fiber communication systems and optical processing applications. However, in order to use the potentiality of these devices, low loss waveguides and efficient coupling are essential.
- ✓ However, in order to use the potentiality of these devices, low loss waveguides and efficient coupling are essential. This leads to the necessity of controlling the mode sizes in the waveguide.
- ✓ For instance, in the design of single mode  $\text{Ti} : \text{LiNbO}_3$  modulators, it is desirable to know the mode sizes to be able to maximize the phase shift along the waveguide, so as to reduce the required voltage and to achieve high efficiency.
- ✓ The schematic diagram of optical waveguide considered in this work is shown in Figure 1.

**Figure 1.** Schematic diagram of the in-diffused  $\text{Ti}:\text{LiNbO}_3$  waveguide.  
( $n_s$  and  $n_g$  are the refractive indices of the substrate and the guide, respectively)



## Analytical method

- A plane wave is assumed to propagate along the y-axis in the strip waveguide, in which the electric field is aligned along the z-axis. Then, from the scalar wave equation:

$$\left( \frac{\partial^2}{\partial x^2} - \beta^2 + \frac{\partial^2}{\partial z^2} + n^2(x, z) k^2 \right) E_z(x, z) = 0$$

where the refractive index  $n(x, z)$ , after diffusion of the titanium, is given in terms of the index  $n_s$  before diffusion, as:

$$n(x, z) = n_s + \Delta n_0 f(x) g(z)$$

It is known that when the diffusion time is sufficiently long, the titanium profiles are given by

$$f(x) = \left\{ \operatorname{erf} \left[ \frac{1}{\sqrt{2D}} \left( x + \frac{W}{2} \right) \right] - \operatorname{erf} \left[ \frac{1}{\sqrt{2D}} \left( x - \frac{W}{2} \right) \right] \right\} / \left\{ 2 \operatorname{erf} \left[ \frac{1}{2\sqrt{2D}} \right] \right\}$$

$$g(z) = \exp \left[ -\frac{1}{2} \left( \frac{z}{D} \right)^2 \right]$$

$$E_z(x, z) = \psi(x) \phi(z)$$

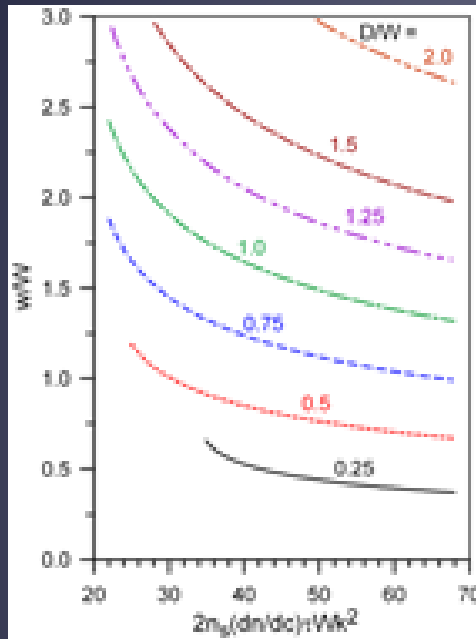


Figure 2 Calculated mode width ratio  $w/W$  as function of  $b$  for various diffusion depth to metal strip width ratio  $D/W$ .

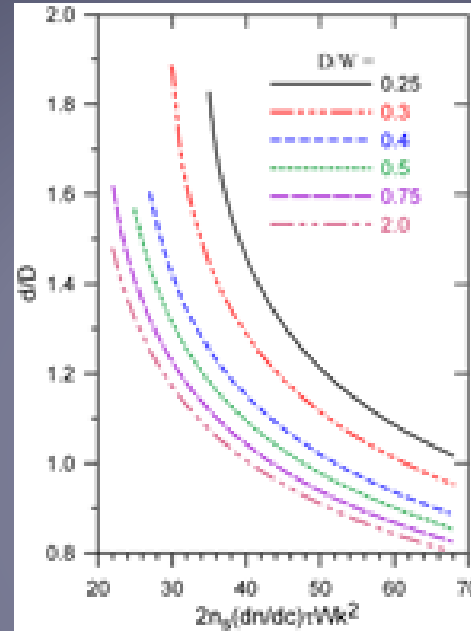


Figure 3 Calculated mode depth ratio  $d/D$  as function of  $b$  for various diffusion depth to metal strip width ratio  $D/W$ .

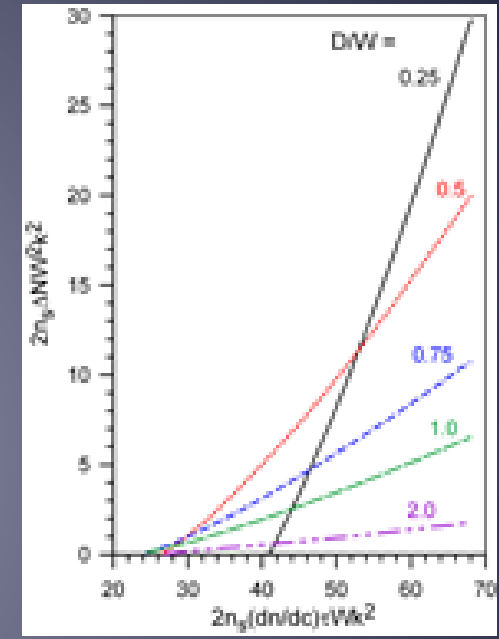


Figure 4 Normalized dispersion curves for the fundamental mode calculated for various diffusion depth to metal strip width ratios  $D/W$ .



The present model and that reported in [4] were also compared to other experimental results [3, 13-14]

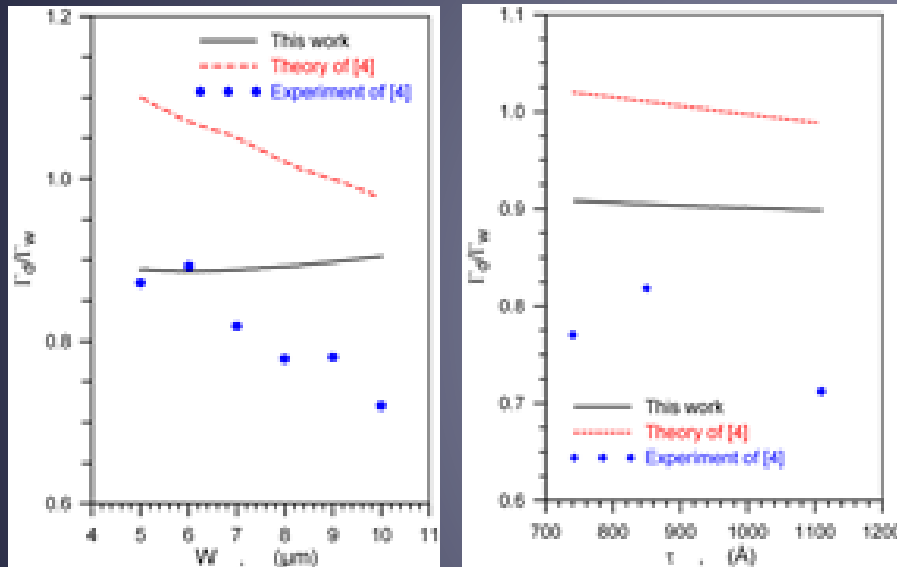


Figure 5 Ratio of mode sizes ( $\Gamma_d/\Gamma_w$ ) as a function of (a) the initial titanium strip width  $W$ ; (b) the initial titanium strip thickness  $\tau$ .

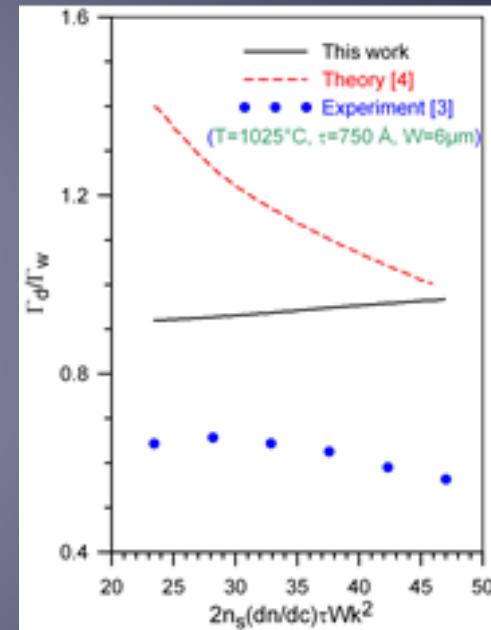


Figure 6 Comparative plots showing the mode size ratio ( $\Gamma_d/\Gamma_w$ ) as function of b.

[3] P.G. Suchoski, R.V. Ramaswamy, IEEE J. Quant. Electron., vol. 23, p. 1673, 1987.

[4] S.K. Korotky, W.J. Minford, L.L. Buhl, M.D. Divino, R.C. Alferness, IEEE J. Quant. Electron., vol. 18, p. 1796, 1982.

[13] L. McCaughan, E.J. Murphy, IEEE J. Quant. Electron., vol. 19, p. 131, 1983.

[14] R.C. Alferness, R.V. Ramaswamy, S.K. Korotky, M.D. Divino, L.L. Buhl, IEEE J. Quant. Electron., vol. 18, p. 1807, 1982.

The idea of the method is to find an approximate solution to a differential equation after a reformulation as an integral identity called a variational form. Instead of trying to satisfy the equation nodes, we decompose the domain by subdomains called finite elements, and this requires satisfying the field equation.

To illustrate the principle of FEM, we take, for instance, the example of the Helmholtz equation and we try to minimize the amount of  $R$  such that:

$$\nabla^2 \phi(x, z) + (k^2 n^2(x, z) - \beta^2) \phi(x, z) = 0$$

$$R = \nabla^2 \phi(x, z) + (k^2 n^2(x, z) - \beta^2) \phi(x, z)$$

A set of  $n$  equations with  $n$  unknowns is obtained, and thus forming a basic system that can be written in the following matrix form:

$$[A_e] \{\phi_e\} = \{b_e\}$$

in which, the matrix  $[A_e]$  is associated with the element. Its coefficients are functions of the coordinates of the element nodes. The components  $\{\phi_e\}$  are unknown variables to the nodes of the electric field of the same element. The vector  $\{b_e\}$  takes into account whatever boundary conditions present on some nodes of the considered element.

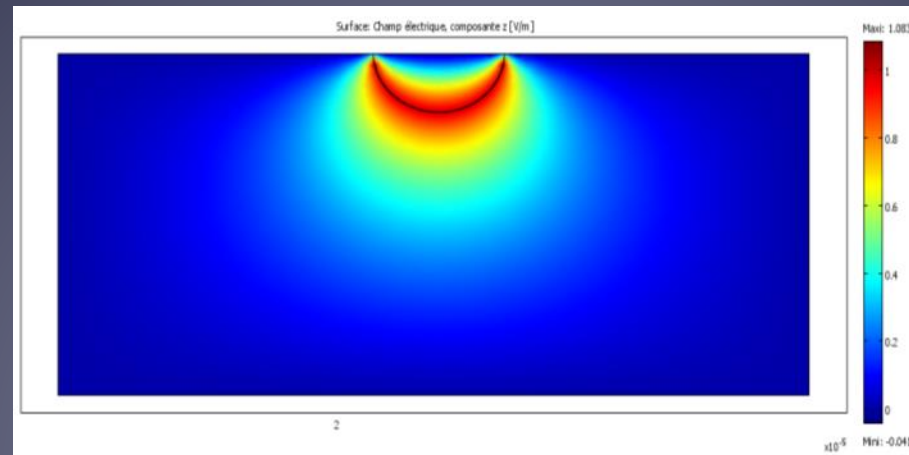


Figure 7. The electric field distribution along the waveguide.

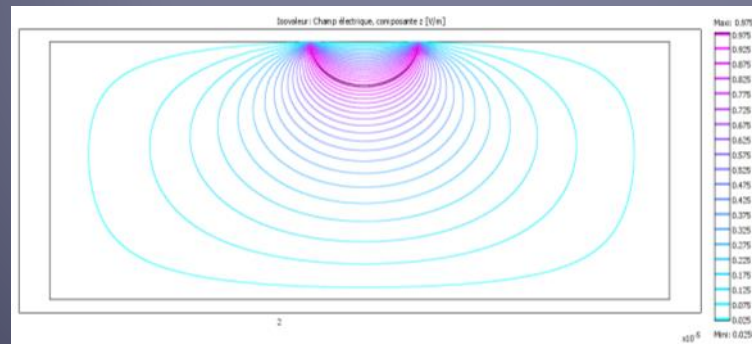


Figure 8. The Equipotential lines of the simulated electrical field distribution inside the waveguide.



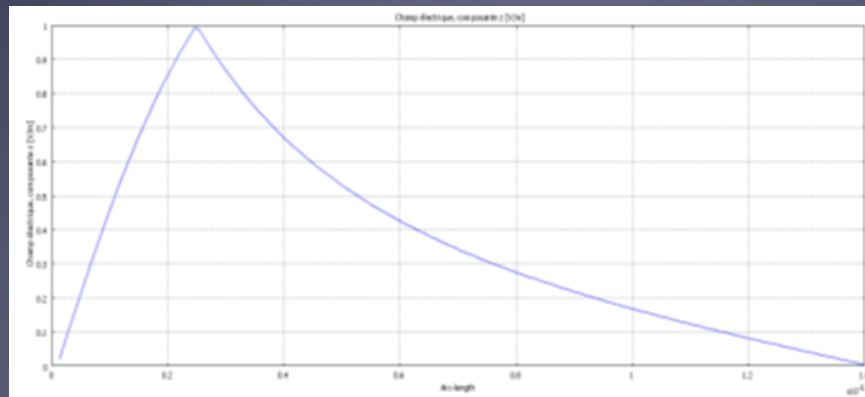


Figure 9. The electric field variation as a function of depth in the substrate.

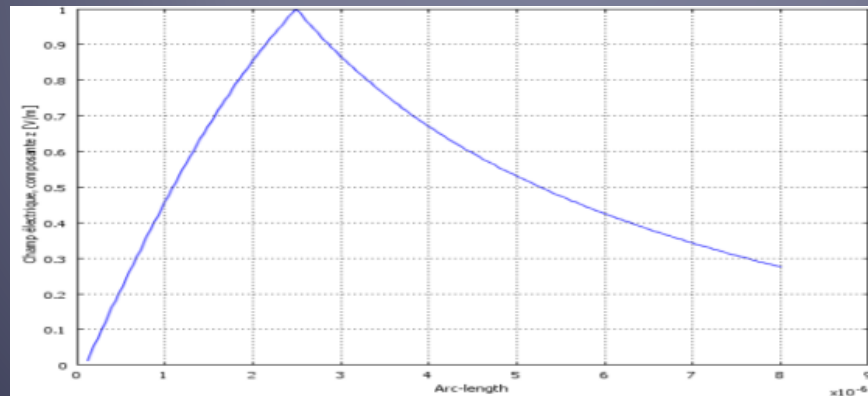


Figure 10. The electric field dependence on depth in the titanium strip.

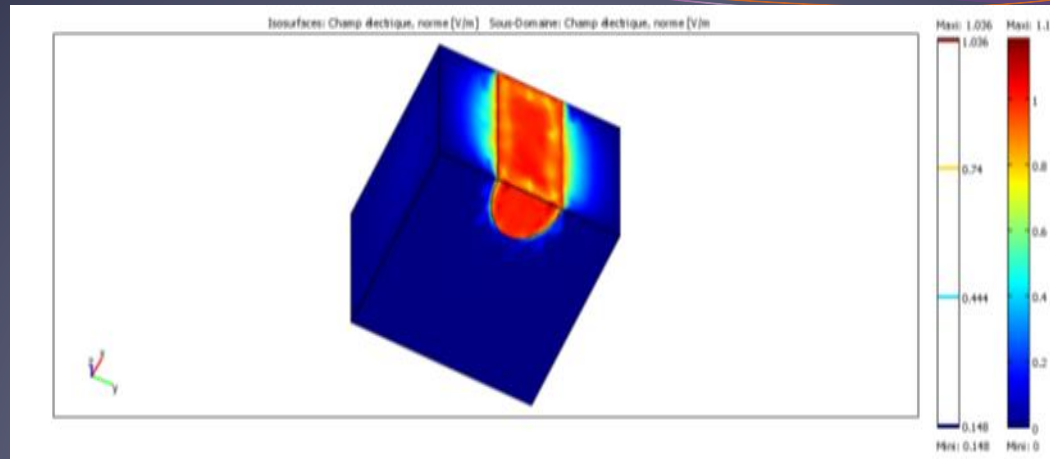


Figure 11. 3-D plot showing the electric field distribution in Ti : LiNbO<sub>3</sub>.

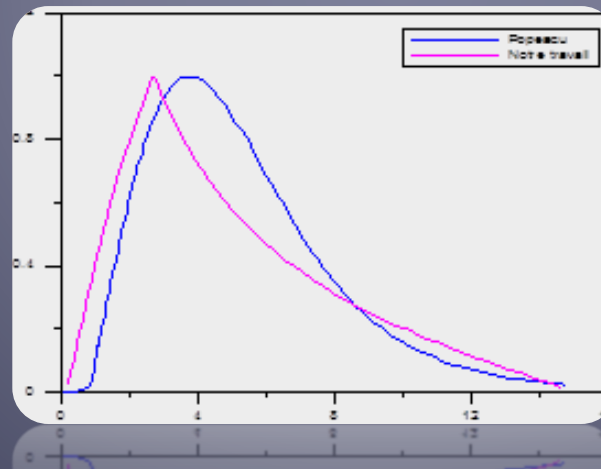


Figure 12. Comparative plot results obtained from our model and that of Popescu [5]

## CONCLUSION

An extension to a previously reported model [4] is given for titanium diffused LiNbO<sub>3</sub> strip waveguide mode sizes. The model is based on the variational method and an expression for the normalized effective index difference is given. The orthogonal spatial mode size parameters are calculated by maximizing the effective index difference. This is done for the two dimensional case by considering the mode size parameters simultaneously. Analytical expressions are given for these mode sizes. Comparison with experimental results published in the literature shows an improved agreement.

we have also presented our results in simulating the properties of light propagation inside an integrated optical waveguide that is based on in-diffused Ti : LiNbO<sub>3</sub> using the finite element method. This work allowed us to establish the fundamentals in order to analyze optical devices that are more complex in structures such as couplers, switches and modulators.



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# Thank You for Your Attention

